

# Joint quantum nondemolition measurements of qubits: beyond the mean-field theory

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We propose an approach to nondestructively detect  $N$  qubits by measuring the transmissions of a dispersively-coupled cavity. By taking into account all the cavity-qubits quantum correlations (i.e., beyond the usual coarse-grained/mean-field approximations), it is revealed that for an unknown normalized  $N$ -qubit state  $|\psi_N\rangle = \sum_{k=0}^{2^N-1} \beta_k |k\rangle_N$ , each detected peak in the cavity transmitted spectra marks one of the basis states  $|k\rangle_N$  and the relative height of such a peak is related to the corresponding superposed-probability  $|\beta_k|^2$ . Our results are able to unambiguously account for the intriguing multi-peak structures of the spectra observed in a very recent circuit-quantum-electrodynamics experiment [Phys. Rev. A **81**, 062325 (2010)] with two superconducting qubits.

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**Introduction.**— It is well-known that the readout of qubits is one of necessary steps in quantum information processing. Phenomenally, the information stored in an unknown  $N$ -qubit quantum state  $|\psi_N\rangle = \sum_{k=0}^{2^N-1} \beta_k |k\rangle_N$  can be partly extracted by directly applying the standard von Neumann projective operation  $\hat{P} = \sum_k |k\rangle_N \langle k|$  to the quantum register [1]. After such a projection, the register is collapsed to one of the computational basis (basis states)  $\{|k\rangle_N = |\sum_{j=1}^N 2^{j-1} \alpha_j\rangle_N = \prod_{j=1}^N |\alpha_j\rangle_N, \alpha = 0, 1\}$  with a probability  $|\beta'_k|^2$ . This is a directly projective measurement (DPM) and the register is detected. Typically, DPM has been utilized to extract the binary quantum information stored in [2], such as trapped ions, Cooper-pair boxes, and the current-biased Josephson junctions, etc.. Essentially, due to the inevitable back actions of the measuring apparatus, the detected  $|\beta'_k|^2$  is always less than its expectable value  $|\beta_k|^2$ . This means that the DPM is not an ideal method to extract the quantum information in an unknown quantum state [3].

can also be utilized to achieve the measurement of the qubits, where another coupled system instead the qubits-selves is detected. Typical examples include, e.g., dc-SQUIDs for the inductively-connected Josephson flux (persistent) qubits [4], optical cavities for the containing atomic qubits [5], and Cooper-pair box for the nanomechanical resonators [6], etc.. A remarkable advantage in the IPM is that the back actions from the detected system could be minimized. If the condition  $[H_N, H_I] = 0$  is satisfied (which means that the disturbance of the detector  $D$  on the qubits is negligible), then the relevant IPM further becomes a quantum nondemolition (QND) measurement [1]. Here,  $H_N$  is the Hamiltonian of the  $N$ -qubit register and  $H_I$  the interaction between it and the detector. Note that the term nondemolition does not imply that the wave function of the register fails to collapse due to the measurement [7]. In fact, for an unknown input state  $|\psi_N\rangle$ , after the QND measurement the  $N$ -qubit register will be automatically collapsed to one of its computational basis  $|k\rangle_N$  with an *ideal* probability  $|\beta_k|^2$ . Thus, QND measurement is a conceptually ideal projective measurement; the successive QND measurements on the same register will give the same result.

As it can be easily detected with the current technique, driven cavity has been widely utilized to achieve the desirable IPM of the dispersively-coupling qubits. Experimentally, it is not difficult to probe the resonance frequency of a driven cavity by detecting the transmitted signals. If the qubits are dispersively coupled to the cavity mode, then the cavity is pulled by the qubits, depending on the state of the qubits. As a consequence, by detecting the shift of the central frequency of the driven cavity mode the QND measurement of the qubits can be achieved. This idea has been experimentally demonstrated by the cavity QED experiments with few qubits, and single basis states (i.e., computational basis) of the atomic and superconducting qubits had been experimentally distin-

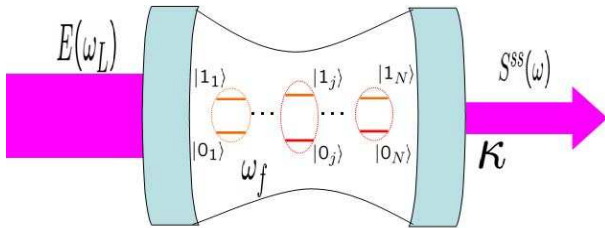


FIG. 1: (Color online) QND measurements of  $N$  qubits (with two levels:  $\{|0_j\rangle, |1_j\rangle\}, j = 1, \dots, N$ ) by measuring the steady-state transmitted spectra  $S_N^{ss}(\omega_L)$  of the dispersively-coupled cavity (with frequency  $\omega_f$  and decay rate  $\kappa$ ) driven by a frequency-controllable external field  $E(\omega_L)$ .

Alternatively, indirectly projective measurements (IPMs)

guished [8, 9]. Next challenge is to completely characterize an unknown  $N$ -qubit state  $|\psi_N\rangle$  by nondestructively measuring an arbitrary superposition of the single basis states with ideal precisions.

Considering the practically-existing dissipation of the detector (i.e., cavity) and also the statistical quantum correlations between it and the  $N$ -qubit quantum register, in this letter we show that an unknown quantum state  $|\psi_N\rangle$  can be effectively nondestructively detected by the realistic QND measurements. Our proposal still works for the mixed states, and thus could be utilized to explain the detected multi-peak structure in the most recent circuit-quantum-electrodynamics (circuit-QED) experiment [12], where the detected qubits could be prepared/decayed at various superpositions of all the possible basis states.

*Generic model.*—The system proposed to nondestructively detect an  $N$ -qubit state is schematized in Fig. 1, wherein  $N$  non-interacting qubits are dispersively coupled to a driven cavity. Certainly, the preparation of the initial state of the qubits and detection of the driven cavity are repeatable. Without loss of generality, we assume that, the coupling strength  $g_j$  and the detuning  $\Delta_j = |\omega_f - \omega_j|$  between the  $j$ th qubit and the cavity, and the detuning  $\Delta_{ij} = |\omega_i - \omega_j|$  between the  $i$ th and  $j$ th qubits satisfy the condition

$$0 < \frac{g_j}{\Delta_j}, \frac{g_i g_j}{\Delta_i \Delta_{ij}}, \frac{g_i g_j}{\Delta_j \Delta_{ij}} \ll 1, \quad i \neq j = 1, 2, \dots, N. \quad (1)$$

This is to realize the dispersive interactions between the qubits and cavity, and to assure that the  $i$ th and  $j$ th qubits are decoupled effectively from each other. Also, the decay rates  $\{\gamma_j\}$  of the qubits should be significantly less than that of the cavity,  $\kappa$ , to ensure that the detected state of the qubits has sufficiently-long lifetime. In fact, all the conditions listed above are practically satisfied in the current typical circuit-QED systems [9].

Under the rotating-wave approximation and in a framework rotating at a frequency  $\omega_L$ , the process for nondestructively measuring the unknown  $N$ -qubit state could be described by the following master equation

$$\begin{aligned} \dot{\rho}_N &= -i[H_N, \rho_N] + \frac{\kappa}{2}(2\hat{a}\rho_N\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\rho_N - \rho_N\hat{a}^\dagger\hat{a}), \\ H_N &= \delta\hat{a}^\dagger\hat{a} + \frac{1}{2}\sum_{j=1}^N\tilde{\omega}_j\sigma_j^z - \hat{a}^\dagger\hat{a}\sum_{j=1}^N\Gamma_j\sigma_j^z + \epsilon(\hat{a}^\dagger + \hat{a}), \end{aligned} \quad (2)$$

with  $\Gamma_j = g_j^2/\Delta_j$  and  $\epsilon$  being the effective strength of the driving. Also,  $\tilde{\omega}_j = \omega_j - \Gamma_j$  is the renormalized transition frequency of the  $j$ th qubit, and  $\delta = \omega_f - \omega_L$  is the detuning between the driving field and the cavity. For the ideal readouts, the dissipation of the qubits is assumed to be negligible. As a consequence, the expectable values of all the related qubit-operators, i.e.,  $\sigma_j^z$ ,  $\sigma_j^z\sigma_k^z$  ( $j \neq k$ ), and the  $N$ -body ones  $\prod_{j=1}^N\sigma_j^z$  are kept unchanged. Our central task is to calculate the steady-state transmitted strength  $S_N^{ss}(\omega_L)$  of the driven cavity. This quantity is essentially proportionate to the number of the steady-state photons in the cavity, i.e.,  $S_N^{ss} = \langle\hat{a}^\dagger\hat{a}\rangle_N^{ss}/\epsilon^2$ , which is determined by the following dy-

namical equation

$$\frac{d\langle\hat{a}^\dagger\hat{a}\rangle_N}{dt} = -\kappa\langle\hat{a}^\dagger\hat{a}\rangle_N - 2\epsilon\text{Im}\langle\hat{a}\rangle_N. \quad (3)$$

Here,  $\langle\hat{a}\rangle_N = \text{Tr}(\hat{a}\rho_N)$  is further determined by

$$\frac{d\langle\hat{a}\rangle_N}{dt} = \left(-i\delta - \frac{\kappa}{2}\right)\langle\hat{a}\rangle_N + i\sum_{j=1}^N\Gamma_j\langle\sigma_j^z\hat{a}\rangle_N - i\epsilon. \quad (4)$$

Neglecting all the statistical quantum correlations between the cavity and qubits, i.e., under the usual coarse-grained (or mean-field) approximation (CGA), see, e.g., [10], we simply have  $\langle\sigma_j^z\hat{a}\rangle_N \approx \langle\sigma_j^z(0)\rangle_N\langle\hat{a}\rangle_N$ . Then, by finding the steady-state solutions to the Eqs. (3-4), one can easily obtain an approximate transmitted spectrum:

$$\tilde{S}_N^{ss}(\omega_L) = \left\{[\omega_L - (\omega_f - \Delta\tilde{\omega}_N)]^2 + \left(\frac{\kappa}{2}\right)^2\right\}^{-1}, \quad (5)$$

with  $\Delta\tilde{\omega}_N = \sum_{j=1}^N\Gamma_j\langle\sigma_j^z(0)\rangle_N$ . This indicates that, compared to the spectrum for the empty cavity (EMC) transmission, the qubits only shift the central frequency with a quantity  $\Delta\tilde{\omega}_N$  and the single-peak shape is unchanged. However, the above CGA is unnecessary and the two-body cavity-qubits correlation functions  $\langle\sigma_j^z\hat{a}\rangle_N$  can be further determined by solving the following dynamical equation

$$\frac{d\langle\sigma_j^z\hat{a}\rangle_N}{dt} = \left(-i\delta - \frac{\kappa}{2}\right)\langle\sigma_j^z\hat{a}\rangle_N - i\epsilon\langle\sigma_j^z\rangle_N + i\sum_{l=1}^N\Gamma_l\langle\sigma_l^z\sigma_j^z\hat{a}\rangle_N. \quad (6)$$

Note that the three-body cavity-qubits correlations  $\langle\sigma_j^z\sigma_l^z\hat{a}\rangle_N$ , introduced above, is related further to the four-body cavity-qubits correlations:  $\langle\sigma_j^z\sigma_l^z\sigma_m^z\hat{a}\rangle_N$ ,  $m = 1, 2, \dots, N$ , etc.. Generally, the  $k$ -body cavity-qubits correlations are related further to the  $(k+1)$ -body cavity-qubits correlations (i.e.,  $k$  qubits correlate simultaneously to the cavity), and thus a series of dynamical equations for these correlations will be induced. Fortunately, due to the fact that  $\sigma_l^z\sigma_m^z = 1$  for  $l = m$ , these equation-chains will be automatically cut off and ended at the  $(N+1)$ -body cavity-qubits correlations. Then, all the interested statistical quantum correlations in these equations can be exactly calculated, and consequently the transmitted spectra can be obtained beyond the usual CGAs. It is emphasized that the spectral distribution  $S_N^{ss}(\omega_L)$  including all the cavity-qubits quantum correlations may reveal  $2^N$  peaks for an  $N$ -qubit state superposed by  $2^N$  basis states. If the detected state is just one of the basis states (not their superposition), then  $S_N^{ss}(\omega_L)$  reduces to  $\tilde{S}_N^{ss}(\omega_L)$  (with a single-peak structure) and the cavity-qubits quantum correlation vanishes.

*Demonstrations with experimentally-existing circuit-QED systems.*—Our generic proposal derived above could be specifically demonstrated with various experimental cavity-qubits systems, typically the circuit-QED one [9, 14]. In this system, the cavity is formed by a coplanar waveguide (of the length at the order of millimeters) and the qubits are generated by the Cooper-pair boxes (CPBs) with controllable Josephson energies. At a sufficiently low temperature (e.g.,

$\leq 20$  mK), the coplanar waveguide works as an ideal superconducting transmission line resonator (i.e., cavity). Experimentally [10], the decay rate (e.g.,  $\kappa = 2\pi \times 1.69$  MHz) of the cavity is about ten times larger than that of the CPB-qubit (e.g.,  $\gamma = 2\pi \times 0.19$  MHz) [11]. Also, by adjusting the external biases, the CPB-qubits could be either coupled to or decoupled from the resonator, and the required initial state preparation and detection can be robustly repeated. For the EMC case, the steady-state solutions to Eqs. (3-4) can be easily obtained and the transmission spectrum reads  $S_0^{ss}(\omega_L) = [(\omega_L - \omega_f)^2 + (\kappa/2)^2]^{-1}$ . Obviously, this is a well-known Lorentzian lineshape [9] centered at  $\omega_f$ , with the half-width  $\kappa$ .

For one qubit case with  $N = 1$ , the steady-state transmission spectrum of the cavity is expressed as

$$S_1^{ss}(\omega_L) = \frac{(\omega_L - \omega_f)^2 - 2(\omega_L - \omega_f)\Gamma_1 Z_1^{(1)}(0) + \Lambda_1}{[(\omega_L - \omega_f)^2 - \Lambda_1^2]^2 + [\kappa(\omega_L - \omega_f)]^2}, \quad (7)$$

with  $\Lambda_1 = \Gamma_1^2 + (\kappa/2)^2$  and  $Z_1^{(1)}(0) = \text{Tr}\{\rho_1(0)\sigma_1^z\} = 2|\beta_1|^2 - 1$ , for the unknown qubit state  $|\psi_1\rangle$ . This is evidently different from the  $\tilde{S}_1^{ss}(\omega_L)$  derived under the usual CGA. Obviously, the spectrum function  $S_1^{ss}(\omega_L)$  predicates that two transmitted peaks could be found in the spectrum. Specifically, using the parameters in the experimental circuit-QED with one CPB-qubit [10]:  $(\omega_f, \omega_0, \kappa, g) = 2\pi \times (6444.2, 4009, 1.69, 134)$  MHz, we plot respectively the spectra  $\tilde{S}_1^{ss}(\omega_L)$  and  $S_1^{ss}(\omega_L)$  in Figs. 2(a) and 2(b) typically for  $|\beta_1|^2 = 0, 0.25, 0.5, 0.75$ , and 1, respectively. For contrasts, the spectrum of the empty cavity (black line) is also plotted in the figures. We make two remarks. Firstly, if the qubit is at one of their basis states (i.e., either  $|0\rangle_1$  or  $|1\rangle_1$ ), then  $\tilde{S}_1^{ss}(\omega_L)$  and  $S_1^{ss}(\omega_L)$  give the same single-peak distribution, which has been experimentally demonstrated [9]. Secondly, if the qubit is prepared beforehand at the superposition of its two basis states, then  $\tilde{S}_1^{ss}(\omega_L)$  shows still the single-peak structure (when  $|\beta_1| = 0.5$ ,  $\tilde{S}_1^{ss}(\omega_L)$  superposes the  $S_0^{ss}(\omega_L)$ ) but  $S_1^{ss}(\omega_L)$  predicts two peaks: the locations of the central frequencies are unchanged, but their relative heights equal respectively to the superposed probabilities of the two basis states. Therefore,  $S_1^{ss}(\omega_L)$  (rather than  $\tilde{S}_1^{ss}(\omega_L)$ ) provides the messages of all the diagonal elements of the density matrix  $\rho_1$ . These predictions should be easily verified with the current experimental technique, once the qubit is input at an arbitrarily-selected superposition state.

Similarly, for  $N = 2$  case [12–14] the steady-state transmitted spectrum can still be analytically obtained:

$$S_2^{ss}(\omega_L) = -\frac{2(AC + BD)}{\kappa(A^2 + B^2)}, \quad (8)$$

with  $A = (\Gamma_1^2 - \Gamma_2^2)^2 + 2[\frac{\kappa^2}{4} - (\omega_L - \omega_f)^2] \sum_{j=1}^2 \Gamma_j^2 + [\frac{\kappa^2}{4} - (\omega_L - \omega_f)^2]^2 - \kappa^2(\omega_L - \omega_f)^2$ ,  $B = -2\kappa(\omega_L - \omega_f)[\sum_{j=1}^2 \Gamma_j^2 + \frac{\kappa^2}{4} - (\omega_L - \omega_f)^2]$ ,  $C = \kappa Z_{1,2}^{(2)}(0)\Gamma_1\Gamma_2 - \kappa(\omega_L - \omega_f) \sum_{j=1}^2 \langle \sigma_j^z(0) \rangle_2 \Gamma_j + \frac{\kappa}{2}[3(\omega_L - \omega_f)^2 - \frac{\kappa^2}{4} - \sum_{j=1}^2 \Gamma_j^2]$ ,  $Z_{1,2}^{(2)}(0) =$

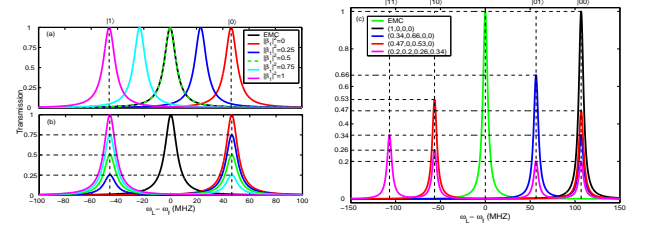


FIG. 2: (Color online) Left: Spectral distributions  $\tilde{S}_1^{ss}(\omega_L)$  (a) and  $S_1^{ss}(\omega_L)$  (b) of the cavity with a single qubit prepared at the state  $|\psi\rangle_1 = \beta_0|0\rangle_1 + \beta_1|1\rangle_1$ , typically for  $|\beta_1|^2 = 0, 0.25, 0.5, 0.75$ , and 1, respectively. Right: Spectral distributions  $S_2^{ss}(\omega_L)$  for the two-qubit register prepared at  $|\psi\rangle_2$  with  $(|\beta_0|^2, |\beta_1|^2, |\beta_2|^2, |\beta_3|^2) = (1, 0, 0, 0), (0.34, 0.66, 0, 0), (0.47, 0, 0.53, 0), (0.2, 0.2, 0.26, 0.34)$ , respectively. In contrast, transmission spectrum of the empty cavity (EMC)  $S_0^{ss}(\omega_L)$  is also shown by the black line.

$\text{Tr}\{\rho_2(0)\sigma_2^z\sigma_2^z\}$ ,  $Z_j^{(2)}(0) = \text{Tr}\{\rho_2(0)\sigma_j^z\}$  and  $D = -2 Z_{1,2}^{(2)}(0)(\omega_L - \omega_f)\Gamma_1\Gamma_2 - \sum_{j=1}^2 Z_j^{(2)}(0)\Gamma_j[\Gamma_j^2 - \Gamma_{j'}^2 + \frac{\kappa^2}{4} - (\omega_L - \omega_f)^2] + (\omega_L - \omega_f)[\sum_{j=1}^2 \Gamma_j^2 + \frac{3\kappa^2}{4} - (\omega_L - \omega_f)^2]$ ,  $j \neq j' = 1, 2$ , respectively. It is seen that, the spectral distribution  $S_2^{ss}(\omega_L)$  may reveal four peaks, but  $\tilde{S}_2^{ss}(\omega_L)$  always shows one-peak structure. The current circuit-QED experiments with two CPB-qubits [12] could be utilized to verify the multi-peak spectral distributions predicted above, once the CPB-qubits are prepared at the superpositions of their basis states. Specifically, with the experimental parameters [12, 13]:  $(\omega_f = \omega_c, \Gamma_1 = \chi^L, \Gamma_2 = \chi^R, \kappa) = 2\pi \times (6.806, 0.013, 0.004, 0.001)$  GHz, one can plot the spectral function  $S_2^{ss}(\omega_L)$  in Fig. 2(c) for the selected two-qubit state  $|\psi\rangle_2$  with:  $(|\beta_0|^2, |\beta_1|^2, |\beta_2|^2, |\beta_3|^2) = (1, 0, 0, 0)$  (black line),  $(0.34, 0.66, 0, 0)$  (blue line),  $(0.47, 0, 0.53, 0)$  (red line), and  $(0.2, 0.2, 0.26, 0.34)$  (pink line), respectively. From these numerical results one can see that, if the two-qubit is prepared at one of the basis states, e.g.,  $|00\rangle$  here, then the transmitted spectrum of the driven cavity shows a single peak. While, if the two-qubit are prepared at the any superposition of their basis states, then the detected spectra should reveal multi-peak structures, i.e., two peaks for black and red lines and four peaks for the pink line.

Generally, if the  $N$ -qubit register is prepared at the superposition of  $M (\leq 2^N - 1)$  basis states, then the cavity could be pulled by  $M$  forms and thus there are  $M$  possible shifts of the cavity resonance frequency. As a consequence, the detected cavity transmitted spectrum will reveal  $M$  peaks; the superposed probability of one of the basis state determines the weight for pulling the cavity and thus the relative height of the corresponding transmitted peak. Therefore, one could presume that the QND measurements of an arbitrary  $N$ -qubit state could be achieved by analyzing the transmission spectra of the dispersively-coupled cavity: from the locations of the central frequencies of the detected peaks, one can determine which basis states  $\{|k\rangle_N\}$  are superposed; and from the relative heights of the corresponding peaks, one can determine the superposed probabilities  $\{|\beta_k|^2\}$ .

*Discussions and Conclusions.*— Our proposal is based on an

important assumption, i.e., each detection should be finished sufficiently-fast such that the influence of the decay of the detected quantum state is negligible. This condition is satisfied in the current circuit-QED experiment [9, 10, 12], wherein each data for recording the transmission event of light through the cavity can be obtained in about 40 ns and the decay time of the detected CPB-qubit is, e.g.,  $T_1 \sim 1\mu s$ . Thus, the predicted multi-peak transmitted spectra could be verified, once the CPB-qubits are prepared at the superposition of various possible basis states (even the mixed ones).

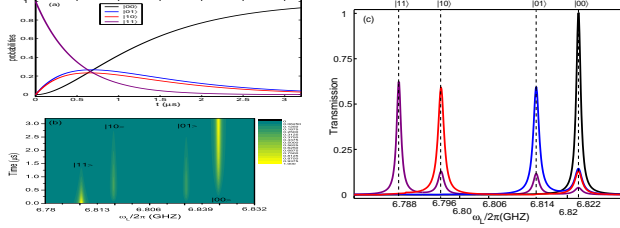


FIG. 3: (Color online) Left: (a) Occupation probabilities of various basis states versus the decay time for the initial state  $|11\rangle$ , (b) Spectral distributions versus the time for the 2-qubit register prepared at the basis state  $|11\rangle$ . Right: (c) Time-averaged transmitted spectra over the time  $\tau = 0.5\mu s$  for different initial states:  $|00\rangle$  (black),  $|01\rangle$  (blue),  $|10\rangle$  (red), and  $|11\rangle$  (pink). Here, the decay rate of the CPB-qubit is taken as  $1/T_{1,j} \sim 1$  MHz.

Immediately, our proposal could be utilized to explain the multi-peak spectra observed in the recent experiment [12]. Strictly speaking, the time-averaged (over a relatively-long time, i.e.,  $0.5\mu s$ , the half of the decay time of the qubit) spectral distributions shown there are not the desirable readouts of the two CPB-qubits, due to the significant decays of the detected states during these relatively-long measuring times for averages. In Fig. 3(a) we typically show how the probabilities of various basis states change with the time from the decays of the excited state  $|11\rangle$  [10, 11]. Clearly, during the decay (e.g., at  $0.5\mu s$ ) the superpositions of the basis states are induced. Thus, based on our proposal beyond the mean-field approximation, the transmitted spectra would reveal a relevant multi-peak structure. Phenomenally, the decay-time dependent steady-state spectrum  $S_2^{ss}(\omega_L, \tau)$  could be ob-

tained by replacing the unchanged expectable values of the qubit-operators in Eq. (8) (i.e.,  $Z_j^{(2)}(0)$  and  $Z_{1,2}^{(2)}(0)$ ) with the decay-time dependent ones [10, 11]. Consequently, the time-dependent spectral distributions (due to the decay of the initial state  $|11\rangle$ ) was simulated in Fig. 3(b), which agrees basically with the corresponding experimental observation (i.e., Fig. 1(E) in Ref. [12]). Thus, except for the non-decayed ground state  $|00\rangle$  (which corresponds certainly to a single transmitted peak), the time-evolution spectra due to the decay of arbitrary excited state, e.g.,  $|10\rangle$  (or  $|01\rangle$ ,  $|11\rangle$ ) would reveal two (or two, four) peaks. By integrating the decay-time dependent steady-state spectrum  $S_2^{ss}(\omega_L, \tau)$ , Fig. 3(b) shows the relevant time-averaged spectra (over the time interval  $\tau = [0, 0.5]\mu s$ ) for these time-evolutions. One can see that the locations of the averaged peaks agree well with the experimental observations [12]. While, the relative heights of the peaks (marking the basis states induced from the decays of the input states) are relatively low. This is an inevitable deduction of multiple QND measurements performed sequentially within the averaged time.

In summary, an efficient approach to implement the QND joint measurements of the  $N$  qubits are proposed by detecting the transmitted spectra through the dispersively-coupled cavity. These measurements are the IPMs of the qubits, and thus the relevant fidelities could be sufficiently high. Remarkably, our proposal is a theory beyond the usual mean-field approximation and thus the statistical quantum correlations between the cavity and qubits are important. In deed, by specifically solving the dynamical equation of the cavity-qubit correlations, e.g., for  $N = 1$  case, one can prove that the lifetime of the qubit-cavity quantum correlation is at the same order of the cavity-self. Therefore, the effects of the cavity-qubit correlations, i.e., the transmitted spectra with multiple peaks, could be verified by inputting the superposed states of the qubits.

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[1] V. B. Braginsky and F. Ya. Khalili, *Quantum Measurement* (Cambridge University Press, 1992); Rev. Mod. Phys. **68**, 1(1996).  
[2] See, e.g., D. Leibfried *et al*, Rev. Mod. Phys. **75**, 281 (2003); G. Johansson *et al*, J. Phys.: Cond. Mat. **18** S901 (2006); K.P. Cooper *et al*, Phys. Rev. Lett. **93**, 180401 (2004).  
[3] T. C. Ralph *et al*, Phys. Rev. A **73**, 012113 (2006); A. N. Korotkov, Phys. Rev. B **78**, 174512 (2008).  
[4] T. Picot *et al*, Phys. Rev. Lett. **105**, 040506 (2010); A. Lupascu, Nature Phys. **3**, 119 (2007).  
[5] J. M. Raimond *et al*, Rev. Mod. Phys. **73**, 565 (2001).  
[6] M. D. LaHaye *et al*, Nature **459**, 960 (2009); L. F. Wei *et al*, Phys. Rev. Lett. **97**, 237201 (2006).

[7] D. Wahyu Utami and A. A. Clerk, Phys. Rev. A **78**, 042323 (2008); L. Chirolli and G. Burkard, Phys. Rev. B **80**, 184509 (2009).  
[8] H. J. Kimble, in *Cavity Quantum Electrodynamics*, edited by P. Bermann (Academic Press, San Diego, 1994); T. Fischer *et al*, Phys. Rev. Lett. **88**, 163002 (2002).  
[9] A. Wallraff *et al*, Nature **431**, 162(2004); Phys. Rev. Lett. **95**, 060501 (2005); A. Blais *et al*, Phys. Rev. A **75**, 032329 (2007).  
[10] R. Bianchetti *et al*, Phys. Rev. A **80**, 043840(2009).  
[11] Taking into account the dissipation  $\{\gamma_j\}$  and and dephasing  $\{\gamma_{\phi,j}\}$  of the qubits, the proposed spectral QND measurements should be described by a more generic master equation:  $\dot{\rho}_N = -i[H_N, \rho_N] + \kappa \mathcal{D}[a]\rho_N +$

$\sum_j \gamma_j \mathcal{D}[\sigma_{-j}] \rho_N + \sum_j \gamma_{\phi,j} \mathcal{D}[\sigma_{z_j}] \rho_N / 2$ , with the dissipation superoperator  $\mathcal{D}[A] \rho_N = A \rho_N A^\dagger - A^\dagger A \rho_N / 2 - \rho_N A^\dagger A / 2$ . Specifically, the expectable values of the qubit-operators are no longer unchanged but determined by the equations (for a two-qubit system):  $d\langle \sigma_i^z \rangle / d\tau = -\gamma_{1,i} (\langle \sigma_i^z \rangle + 1)$ ,  $i = 1, 2$ , and  $d\langle \sigma_1^z \sigma_2^z \rangle / d\tau = -(\sum_{i=1,2} \gamma_{1,i}) \langle \sigma_1^z \sigma_2^z \rangle - \sum_{i=1,2} \gamma_{1,i} \langle \sigma_i^z \rangle$ . For a single detection with the duration  $T_e \sim 40\text{ns}$  and the qubit decay rate  $\gamma_{1,j}^{-1} \sim 1\mu\text{s}$ , the decay effects is really negli-

gible, i.e., the decay factor  $\exp(-\gamma_{1,j} T_e) \sim 1$ . While, if the measurements went on a relatively-long time, e.g.,  $\tau \sim 0.5\mu\text{s}$  in Ref. [12], to perform several QND-detections for time averages, then the decay effects should be taken into account.

[12] J.M. Chow *et al*, Phys. Rev. A **81**, 062325 (2010); cond-mat/0908.1955.

[13] L. DiCarlo *et al*, Nature **460**, 240 (2009).

[14] S. Filipp *et. al.*, Phys. Rev. Lett. **102**, 200402 (2009).